A piece of contemporary art consists of a list of 896 consecutive integers, each in a different color. **SCORE: _____ / 4 PTS**

[a] If 1793 is the smallest integer, what is the largest integer ? $n-1793+1=896 \Rightarrow n=896+1793-1=2688$

[b] If 1793 is the largest integer, what is the smallest integer ? $1793-m+1=896 \Rightarrow m=1793-896+1=898$

E-mail IDs at a certain school are a sequence of 3 letters (selected from A to Z) followed by 4 digits (selected from 0 to 9). For example, one such ID would be $BSL3075$. **SCORE: _____ / 10 PTS**

[a] How many different e-mail IDs are possible ?

Choose the first letter 26 ways
Choose the second letter 26 ways
Choose the third letter 26 ways
Choose the first digit 10 ways
Choose the second digit 10 ways
Choose the third digit 10 ways
Choose the fourth digit 10 ways TOTAL = $26^3 \times 10^4$

[b] How many different e-mail IDs do **<u>NOT</u>** contain any repeated letters ?
(A repeated letter is a letter that appears more than once, not necessarily in consecutive positions.)

Choose the first letter 26 ways
Choose the second letter (not the same as the first) 25 ways
Choose the third letter (not the same as the first two) 24 ways
Choose the first digit 10 ways
Choose the second digit 10 ways
Choose the third digit 10 ways
Choose the fourth digit 10 ways TOTAL = $26 \times 25 \times 24 \times 10^4$

[c] How many different e-mail IDs contain at least one repeated digit ?

Number of e-mail IDs that do NOT contain at least one repeated digit
Choose the first letter 26 ways
Choose the second letter 26 ways
Choose the third letter 26 ways
Choose the first digit 10 ways
Choose the second digit (not the same as the first) 9 ways
Choose the third digit (not the same as the first two) 8 ways
Choose the fourth digit (not the same as the first three) 7 ways SUBTOTAL = $26^3 \times 10 \times 9 \times 8 \times 7$

Number of e-mail IDs that contain at least one repeated digit = $26^3 \times 10^4 - 26^3 \times 10 \times 9 \times 8 \times 7$

[d] How many different e-mail IDs do **<u>NOT</u>** contain the letter X nor any repeated letters ?

Choose the first letter (not X) 25 ways
Choose the second letter (not X nor the same as the first) 24 ways
Choose the third letter (not X nor the same as the first two) 23 ways
Choose the first digit 10 ways
Choose the second digit 10 ways
Choose the third digit 10 ways
Choose the fourth digit 10 ways TOTAL = $25 \times 24 \times 23 \times 10^4$

You must arrange the 11 letters of the word $DOCUMENTARY$ in a row. **SCORE: _____ / 13 PTS**
Each letter will appear exactly once in your arrangement.

[a] How many ways can the letters be arranged so that the word $MONEY$ appears ?
(That is, the letters $MONEY$ are in consecutive positions in that left-to-right order.)

Treat $MONEY$ as one giant letter, and permute along with the other 6 letters (total 7 "letters")
TOTAL = $7!$

[b] How many ways can the letters be arranged so that the word $CARD$ does **<u>NOT</u>** appear ?

Number of arrangements with no restrictions = $11!$

Number of arrangements in which $CARD$ **<u>DOES</u> appear**
Treat $CARD$ as one giant letter, and permute along with the other 7 letters (total 8 "letters")
SUBTOTAL = $8!$

Number of arrangements in which $CARD$ **does <u>NOT</u> appear**
TOTAL = $11! - 8!$

[c] How many ways can the letters be arranged so that the words $MONEY$ and $CARD$ both appear ?

Treat $MONEY$ and $CARD$ as two giant letters, and permute along with the other 2 letters (total 4 "letters")
TOTAL = $4!$

[d] How many ways can the letters be arranged so that neither the words $MONEY$ nor $CARD$ appear ?
(HINT: This is **<u>NOT</u>** the "complement" of [c].)

Let $M = \{$arrangements in which $MONEY$ appears$\}$ $|M| = 7!$
Let $C = \{$arrangements in which $CARD$ appears$\}$ $|C| = 8!$
$M \cap C = \{$arrangements in which $MONEY$ and $CARD$ both appear$\}$ $|M \cap C| = 4!$
$M \cup C = \{$arrangements in which $MONEY$ or $CARD$ or both appear$\}$ $|M \cup C|$
$= |M| + |C| - |M \cap C|$
$= 7! + 8! - 4!$

$(M \cup C)^C = \{$arrangements in which neither $MONEY$ nor $CARD$ appear$\}$
$= 11! - (7! + 8! - 4!)$ or $= 11! - 7! - 8! + 4!$

The film club has 42 members. 18 of them have seen Holy Motors. 20 of them have seen Skyfall. **SCORE: _____ / 8 PTS**
7 of them have seen both Holy Motors & Skyfall. 12 of them have seen Anna Karenina. 9 of them have seen both Skyfall & Anna Karenina. 4 of them have seen both Holy Motors & Anna Karenina. If 9 of them have not seen any of the three movies, how many have seen all three movies ? Show proper algebraic work, including proper set notation. **Venn diagrams & trial-and-error are NOT acceptable.**

Let $U = \{$members of film club$\}$
Let $H = \{$members of film club who have seen Holy Motors$\}$
Let $S = \{$members of film club who have seen Skyfall$\}$
Let $A = \{$members of film club who have seen Anna Karenina$\}$
$H \cap S = \{$members of film club who have seen both Holy Motors & Skyfall$\}$
$H \cap A = \{$members of film club who have seen both Holy Motors & Anna Karenina$\}$
$S \cap A = \{$members of film club who have seen both Skyfall & Anna Karenina$\}$
$H \cap S \cap A = \{$members of film club who have seen all three movies$\}$
$H \cup S \cup A = \{$members of film club who have seen one, two or all three movies$\}$
$(H \cup S \cup A)^C = \{$members of film club who have not seen any of the three movies$\}$

$$|U| = |H \cup S \cup A| + |(H \cup S \cup A)^C|$$
$$42 = |H \cup S \cup A| + 9$$
$$|H \cup S \cup A| = 33$$

$$|H \cup S \cup A| = |H| + |S| + |A| - |H \cap S| - |H \cap A| - |S \cap A| + |H \cap S \cap A|$$
$$33 = 18 + 20 + 12 - 7 - 4 - 9 + |H \cap S \cap A|$$
$$|H \cap S \cap A| = 3$$

3 members have seen all three movies